

Online Linear Learning

John Langford, Large Scale Machine Learning Class,
January 29

To follow along:

```
git clone
```

```
git://github.com/JohnLangford/vowpal_wabbit.git
```

```
wget
```

```
http://hunch.net/~jl/rcv1.train.txt.gz
```

(Post presentation updated version)

Linear Learning

Features: a vector $x \in \mathbb{R}^n$

Label: $y \in \mathbb{R}$

Goal: Learn $w \in \mathbb{R}^n$ such that $\hat{y}_w(x) = \sum_i w_i x_i$ is close to y .

Linear Learning

Features: a vector $x \in \mathbb{R}^n$

Label: $y \in \{-1, 1\}$

Goal: Learn $w \in \mathbb{R}^n$ such that $\hat{y}_w(x) = \sum_i w_i x_i$ is close to y .

Online Linear Learning

Start with $\forall i : w_i = 0$

Repeatedly:

- 1 Get **features** $x \in \mathbb{R}^n$.
- 2 Make linear **prediction** $\hat{y}_w(x) = \sum_i w_i x_i$.
- 3 Observe **label** $y \in [0, 1]$.
- 4 **Update** weights so $\hat{y}_w(x)$ is closer to y .

Online Linear Learning

Start with $\forall i : w_i = 0$

Repeatedly:

- 1 Get features $x \in \mathbb{R}^n$.
- 2 Make logistic prediction $\hat{y}_w(x) = \frac{1}{1+e^{-\sum_i w_i x_i}}$.
- 3 Observe label $y \in [0, 1]$.
- 4 Update weights so $\hat{y}_w(x)$ is closer to y .

Online Linear Learning

Start with $\forall i : w_i = 0$

Repeatedly:

- 1 Get **features** $x \in \mathbb{R}^n$.
- 2 Make linear **prediction** $\hat{y}_w(x) = \sum_i w_i x_i$.
- 3 Observe **label** $y \in [0, 1]$.
- 4 **Update** weights so $\hat{y}_w(x)$ is closer to y .

Online Linear Learning

Start with $\forall i : w_i = 0$

Repeatedly:

- 1 Get **features** $x \in \mathbb{R}^n$.
- 2 Make linear **prediction** $\hat{y}_w(x) = \sum_i w_i x_i$.
- 3 Observe **label** $y \in [0, 1]$.
- 4 **Update** weights so $\hat{y}_w(x)$ is closer to y .
Example: $w_i \leftarrow w_i + \eta(y - \hat{y})x_i$.

An Example: The RCV1 dataset

Pick whether a document is in category CCAT or not.

Dataset size:

781K examples

60M nonzero features

1.1G bytes

Format: label | sparse features ...

An Example: The RCV1 dataset

Pick whether a document is in category CCAT or not.

Dataset size:

781K examples

60M nonzero features

1.1G bytes

Format: label | sparse features ...

1 | 13:3.9656971e-02 24:3.4781646e-02 ...

An Example: The RCV1 dataset

Pick whether a document is in category CCAT or not.

Dataset size:

781K examples

60M nonzero features

1.1G bytes

Format: label | sparse features ...

1 | 13:3.9656971e-02 24:3.4781646e-02 ...

which corresponds to:

1 | tuesday year ...

An Example: The RCV1 dataset

Pick whether a document is in category CCAT or not.

Dataset size:

781K examples

60M nonzero features

1.1G bytes

Format: label | sparse features ...

1 | 13:3.9656971e-02 24:3.4781646e-02 ...

which corresponds to:

1 | tuesday year ...

command: `time vw -sgd rcv1.train.txt -c`

takes 1-3 seconds on my laptop.

Reasons for Online Learning

- 1 **Fast convergence** to a good predictor
- 2 It's **RAM efficient**. You need store only one example in RAM rather than all of them. \Rightarrow Entirely new scales of data are possible.
- 3 **Online Learning algorithm = Online Optimization Algorithm**. Online Learning Algorithms \Rightarrow the ability to solve entirely new categories of applications.
- 4 Online Learning = ability to deal with **drifting distributions**.

Defining updates

Defining updates

- 1 Define a **loss function** $L(\hat{y}_w(x), y)$.

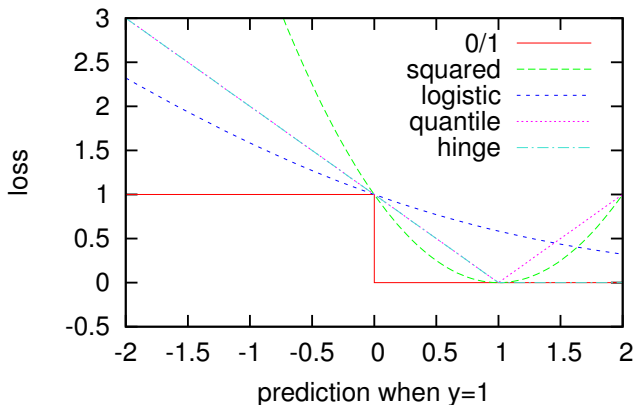
Defining updates

- 1 Define a **loss function** $L(\hat{y}_w(x), y)$.
- 2 **Update** according to $w_i \leftarrow w_i - \eta \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$.
Here η is the **learning rate**.

Defining updates

- 1 Define a **loss function** $L(\hat{y}_w(x), y)$.
 - 2 **Update** according to $w_i \leftarrow w_i - \eta \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$.
- Here η is the **learning rate**.

common loss functions



Know your loss function semantics

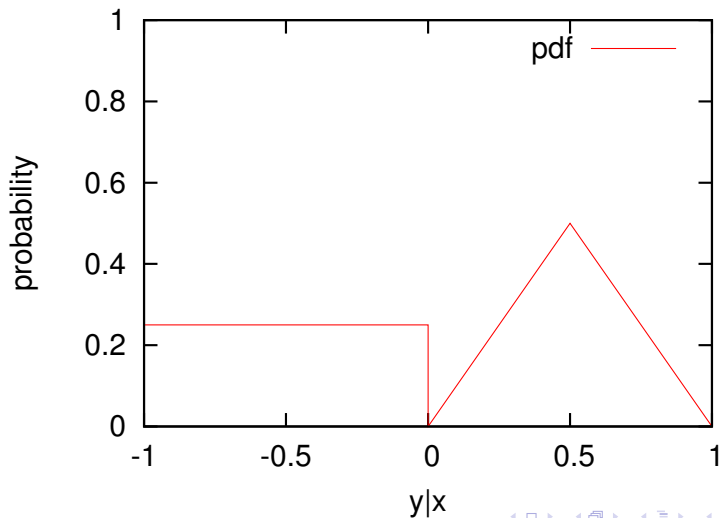
- 1 What is a typical price for a house?
- 2 What is the expected return on a stock?
- 3 What is the probability of a click on an ad?
- 4 Is the digit a 1?
- 5 What do you really care about?

Know your loss function semantics

- 1 What is a typical price for a house?
quantile: minimizer = median
- 2 What is the expected return on a stock?
squared: minimizer = expectation
- 3 What is the probability of a click on an ad?
logistic: minimizer = probability
- 4 Is the digit a 1?
hinge: closest 0/1 approximation
- 5 What do you really care about?
often 0/1

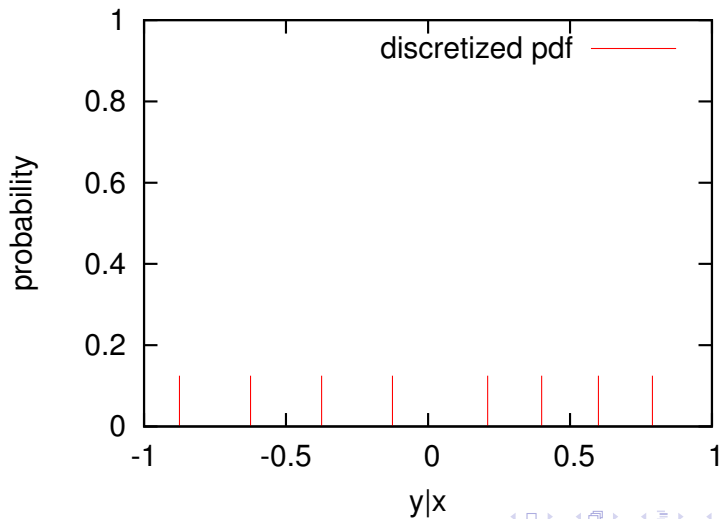
The proof for quantile regression

Consider conditional probability distribution $D(y|x)$.



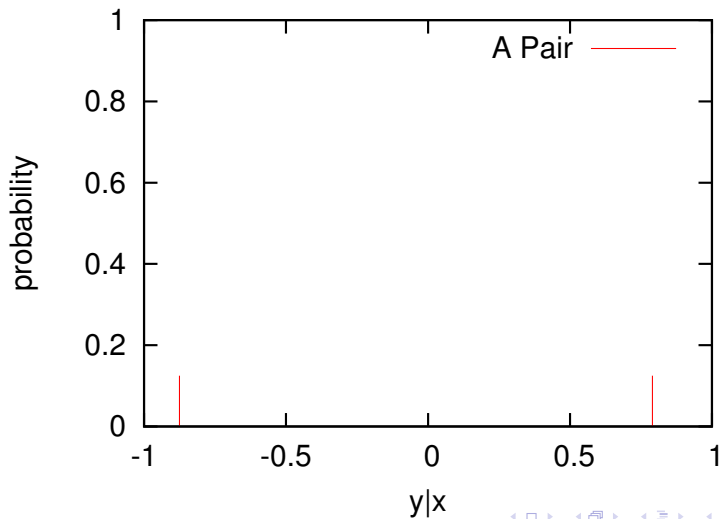
The proof for quantile regression

Discretize it into equal mass bins.



The proof for quantile regression

Where is absolute value minimized?



The proof for quantile regression

note that minimizer = discrete median and take the limit as bin mass goes to 0.

rcv1 with different loss functions

All the common loss functions are sound for binary classification, so which is best is an empirical choice.

```
vw -sgd rcv1.train.txt -c -loss_function  
hinge
```

```
vw -sgd rcv1.train.txt -c -loss_function  
logistic
```

```
vw -sgd rcv1.train.txt -c -loss_function  
quantile
```

How do you know when you succeed?

Standard answer: train on train set, test on test set.
But can we do better?

How do you know when you succeed?

Standard answer: train on train set, test on test set.
But can we do better?

Progressive Validation

On timestep t let $l_t = L(\hat{y}_{w_t}(x_t), y_t)$.

Report loss $L = E_t l_t$.

How do you know when you succeed?

Progressive Validation

On timestep t let $l_t = L(\hat{y}_{w_t}(x_t), y_t)$.

Report loss $L = E_t l_t$.

PV analysis

Let D be a distribution over x, y . Let

$$\bar{l}_t = E_{(x,y) \sim D} L(\hat{y}_{w_t}(x), y)$$

Theorem: For all probability distributions $D(x, y)$, for all online learning algorithms, with probability $1 - \delta$:

$$|L - E_t \bar{l}_t| \leq \sqrt{\frac{\ln 2/\delta}{2T}}$$

rcv1 with different loss functions

All the common loss functions are sound for binary classification, so which is best is an empirical choice.

```
vw -sgd rcv1.train.txt -c -loss_function  
hinge -binary
```

```
vw -sgd rcv1.train.txt -c -loss_function  
logistic -binary
```

```
vw -sgd rcv1.train.txt -c -loss_function  
quantile -binary
```

rcv1 with different loss functions

All the common loss functions are sound for binary classification, so which is best is an empirical choice.

```
vw -sgd rcv1.train.txt -c -loss_function  
hinge -binary
```

```
vw -sgd rcv1.train.txt -c -loss_function  
logistic -binary
```

```
vw -sgd rcv1.train.txt -c -loss_function  
quantile -binary
```

Progressive validation often does not replace train/test discipline, but it can greatly aid empirical testing.

Part II, advanced updates

- 1 Importance weight invariance
- 2 Adaptive updates
- 3 Normalized updates

Learning with importance weights

A common scenario: you need to do classification but one choice is more expensive than the other.

An example: In spam detection, predicting **nonspam as spam** is worse than **spam as nonspam**.

Learning with importance weights

A common scenario: you need to do classification but one choice is more expensive than the other.

An example: In spam detection, predicting **nonspam as spam** is worse than **spam as nonspam**.

Let's say an example is λ times more important than a typical example.

How do you modify the update to use λ ?

Learning with importance weights

A common scenario: you need to do classification but one choice is more expensive than the other.

An example: In spam detection, predicting **nonspam as spam** is worse than **spam as nonspam**.

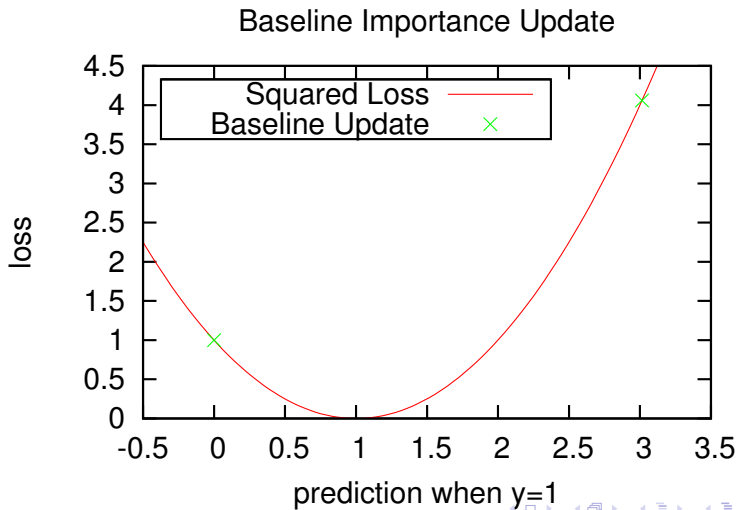
Let's say an example is l times more important than a typical example.

How do you modify the update to use l ?

The baseline approach: $w_i \leftarrow w_i - \eta l \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$.

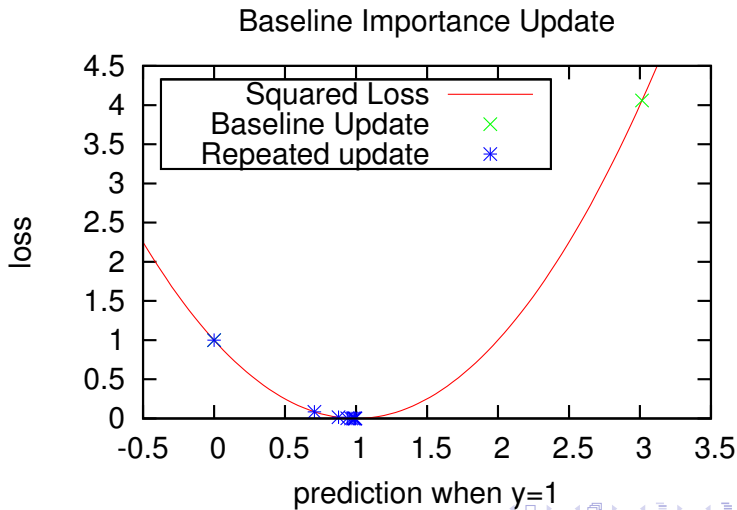
Dealing with the importance weights

$w_i \leftarrow w_i - \eta \left| \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i} \right|$ performs poorly.



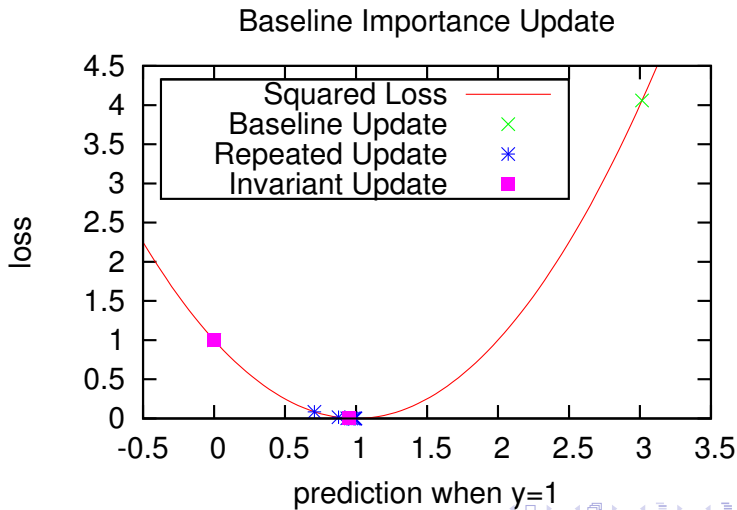
Dealing with the importance weights

A better approach: $w_i \leftarrow w_i - \eta \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$ / times

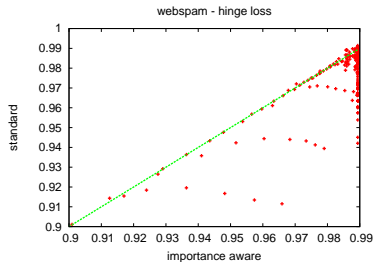
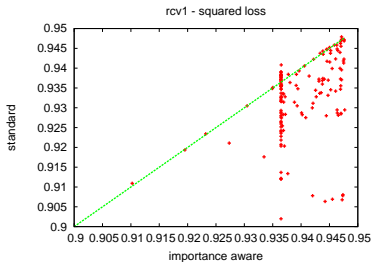
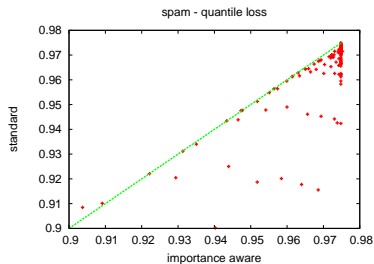
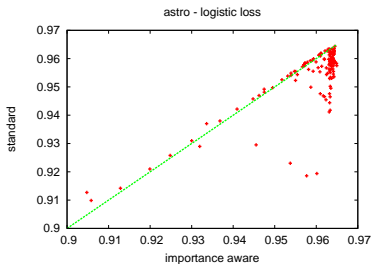


Dealing with the importance weights

An even better approach: $w_i \leftarrow w_i - s(\eta l) \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$



Robust results for unweighted problems



rcv1 with an invariant update

```
vw rcv1.train.txt -c -binary -invariant
```

Performs slightly worse with the default learning rate, but much more robust to learning rate choice.

Adaptive Learning

Learning rates must decay to converge, but how?

Adaptive Learning

Learning rates must decay to converge, but how?
Common answer: $\eta_t = 1/t^{0.5}$ or $\eta_t = 1/t$.

Adaptive Learning

Learning rates must decay to converge, but how?

Common answer: $\eta_t = 1/t^{0.5}$ or $\eta_t = 1/t$.

Better answer: t , let $g_{it} = \frac{\partial L(\hat{y}_w(x_t), y_t)}{\partial w_i}$.

New update rule: $w_i \leftarrow w_{it} - \eta \frac{g_{it}}{\sqrt{\sum_{t'=1}^t g_{it'}^2}}$

Adaptive Learning

Learning rates must decay to converge, but how?

Common answer: $\eta_t = 1/t^{0.5}$ or $\eta_t = 1/t$.

Better answer: t , let $g_{it} = \frac{\partial L(\hat{y}_w(x_t), y_t)}{\partial w_i}$.

New update rule: $w_i \leftarrow w_{it} - \eta \frac{g_{it}}{\sqrt{\sum_{t'=1}^t g_{it'}^2}}$

Common features stabilize quickly. Rare features can have large updates.

Adaptive Learning example

```
vw rcv1.train.txt -c -binary -adaptive
```

Slightly worse. Adding in `-invariant -l 1` helps.

Dimensional Correction

g_{it} for squared loss = $2(\hat{y}_w(x) - y)x_i$ so update is

$$w_i \leftarrow w_i - Cx_i$$

The same form occurs for all linear updates.

Dimensional Correction

g_{it} for squared loss = $2(\hat{y}_w(x) - y)x_i$ so update is

$$w_i \leftarrow w_i - Cx_i$$

The same form occurs for all linear updates.
Intrinsic problems! Doubling x_i implies halving w_i to get the same prediction.
 \Rightarrow Update rule has mixed units!

A standard solution: Gaussian sphering

For each feature x_i compute:

empirical mean $\mu_i = E_t x_{it}$

empirical standard deviation $\sigma_i = \sqrt{E_t (x_{it} - \mu_i)^2}$

Let $x'_i \leftarrow \frac{x_i - \mu_i}{\sigma_i}$.

A standard solution: Gaussian sphering

For each feature x_i compute:

empirical mean $\mu_i = E_t x_{it}$

empirical standard deviation $\sigma_i = \sqrt{E_t (x_{it} - \mu_i)^2}$

Let $x'_j \leftarrow \frac{x_j - \mu_j}{\sigma_j}$.

Problems:

- 1 Lose online.
- 2 RCV1 becomes a factor of 500 larger.

A scale-free update

NG(learning_rate η)

- 1 Initially $w_i = 0$, $s_i = 0$, $N = 0$
- 2 For each timestep t observe example (x, y)
 - 1 For each i , if $|x_i| > s_i$
 - 1 $w_i \leftarrow \frac{w_i s_i^2}{|x_i|^2}$
 - 2 $s_i \leftarrow |x_i|$
 - 2 $\hat{y} = \sum_i w_i x_i$
 - 3 $N \leftarrow N + \sum_i \frac{x_i^2}{s_i^2}$
 - 4 For each i ,
 - 1 $w_i \leftarrow w_i - \eta \sqrt{\frac{N}{t} \frac{1}{s_i^2}} \frac{\partial L(\hat{y}, y)}{\partial w_i}$

A scale-free update

NG(learning_rate η)

- 1 Initially $w_i = 0$, $s_i = 0$, $N = 0$
- 2 For each timestep t observe example (x, y)
 - 1 For each i , if $|x_i| > s_i$
 - 1 Renormalize w_i for new scale
 - 2 Adjust Scale
 - 2 $\hat{y} = \sum_i w_i x_i$
 - 3 Adjust global scale
 - 4 For each i ,
 - 1 $w_i \leftarrow w_i - \eta \sqrt{\frac{N}{t} \frac{1}{s_i^2}} \frac{\partial L(\hat{y}, y)}{\partial w_i}$

In combination

An adaptive, scale-free, importance invariant update rule.

```
vw rcv1.train.txt -c -binary
```

References

[RCV1 example] Leon Bottou, Stochastic Gradient Descent, 2007.

[VW] Vowpal Wabbit project,
<http://hunch.net/~vw>, 2007-2012.

[Quantile Regression] Roger Koenker, Quantile Regression, Econometric Society Monograph Series, Cambridge University Press, 2005.

[Classification Consistency] Ambuj Tewari and Peter L. Bartlett. On the consistency of multiclass classification methods. COLT, 2005.

References

- [[Progressive Validation I](#)] Avrim Blum, Adam Kalai, and John Langford Beating the Holdout: Bounds for KFold and Progressive Cross-Validation. COLT99.
- [[Progressive Validation II](#)] N. Cesa-Bianchi, A. Conconi, and C. Gentile On the generalization ability of on-line learning algorithms IEEE Transactions on Information Theory, 50(9):2050-2057, 2004.
- [[Importance Aware Updates](#)] Nikos Karampatziakis and John Langford, Importance Weight Aware Gradient Updates UAI 2010.
- [[Online Convex Programming](#)] Martin Zinkevich, Online convex programming and generalized infinitesimal gradient ascent, ICML 2003.

References

[Adaptive Updates I] John Duchi, Elad Hazan, and Yoram Singer, Adaptive Subgradient Methods for Online Learning and Stochastic Optimization, COLT 2010 & JMLR 2011.

[Adaptive Updates II] H. Brendan McMahan, Matthew Streeter, Adaptive Bound Optimization for Online Convex Optimization, COLT 2010.

[Scale invariant updates] Forthcoming.