Nonstationary Policy Evaluation

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{ With help from many }

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A Reminder: The Contextual Bandit Setting

For $t = 1, \ldots, T$:

1. The world produces some context $x_t \in X$
2. The learner chooses an action $a_t \in \{1, \ldots, K\}$
3. The world reacts with reward $r_t(a_t) \in [0, 1]$

Goal: Efficiently competing with a large reference class of possible policies $\Pi = \{\pi : X \to \{1, ..., K\}\}$:

$$\text{Regret} = \max_{\pi \in \Pi} \sum_{t=1}^{T} r_t(\pi(x_t)) - \sum_{t=1}^{T} r_t(a_t)$$

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Which combined explore/exploit algorithm is best for your setting?

A Rejection Sampling approach

Rejection_Sampler(policy π, events $(\vec{x}, a, r, p)^T$)
Let $h = \emptyset$ a history, $R = 0$
For each event $(\vec{x}, a, r, p)$

1. If $\pi(h, \vec{x}) = a$
2. then with probability $\frac{p_{\min}}{p}$
    1. $h \leftarrow h \cup (\vec{x}, a, r)$
    2. $R \leftarrow R + r$

Return $R/|h|$

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Theorem: For all history lengths T, For all nonstationary policy π, and all IID worlds D, the probability of a simulated history of length T = the probability of the same history of length T in the real world.

The Master Evaluator

Eval(policy π, events $(\vec{x}, a, r, p)^T$, quantile ρ, bound b)

Let $h = \emptyset$, $R = 0$, $C = 0$, $Q = \emptyset$, c = b

For each event $(\vec{x}, a, r, p)$

1. $R \leftarrow R + c\left(\frac{\pi(a|x,h)}{p}(r - \hat{r}(x,a)) + \sum_{a'} \pi(a'|x,h)\hat{r}(x,a')\right)$
2. $C \leftarrow C + c$
3. $Q \leftarrow Q \cup \left\{\frac{p}{\pi(a|x,h)}\right\}$
4. With probability $\frac{c\pi(a|x,h)}{p}$:
   1. $h \leftarrow h + (x, a, r)$
   2. $c \leftarrow \min\{b,\ \rho\text{-th quantile of } Q\}$

Return R/C

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Incorporates Double Robust + Nonstationary evaluation.
Theorem: Introduces bounded bias + much more efficient.
Empirically, an order of magnitude better for nonstationary eval.

Bibliography

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